

CP violation induced by the double resonance for pure annihilation decay process in Perturbative QCD

Gang Lü^{1*}, Ye Lu^{2†}, Sheng-Tao Li¹ and Yu-Ting Wang¹

¹*College of Science, Henan University of Technology, Zhengzhou 450001, China*

²*Department of Physics, Guangxi Normal University, Guilin 541004, China*

In Perturbative QCD (PQCD) approach we study the direct *CP* violation in the pure annihilation decay process of $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ induced by the ρ and ω double resonance effect. Generally, the *CP* violation is small in the pure annihilation type decay process. However, we find that the *CP* violation can be enhanced by double $\rho - \omega$ interference when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. For the decay process of $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$, the maximum *CP* violation can reach 28.64%.

PACS numbers: 11.30.Er, 12.39.-x, 13.20.He, 12.15.Hh

I. INTRODUCTION

CP violation is an important area in searching new physics signals beyond the standard model(SM). It is generally believed that the *B* meson system provides rich information about *CP* violation. The theoretical work has been done in this direction in the past few years. *CP* violation arises from the weak phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2] in SM. Meanwhile, it is remarkable that *CP* violation can still be produced by the interference effects between the tree and penguin amplitudes. Since the kinematic suppression, the strong phase associated with long distance rescattering is generally neglected during the past decades. Recently, the LHCb Collaboration found the large *CP* violation in the three-body decay channels of $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ and $B^\pm \rightarrow K^\pm\pi^+\pi^-$ [3–5]. Hence, the nonleptonic *B* meson decay from the three-body and four-body decay channels has been become an important area in searching for *CP* violation.

A mixing between the *u* and *d* flavor leads to the breaking of isospin symmetry for the $\rho - \omega$ system. The chiral dynamics has been shown restore the isospin symmetry [6]. The $\rho - \omega$ mixing matrix element $\tilde{\Pi}_{\rho\omega}(s)$ gives rise to isospin violation, where *s* is the Mandelstam variable. The magnitude has been extracted by the pion form factor through the cross section of $e^+e^- \rightarrow \pi^+\pi^-$. We can separate the $\tilde{\Pi}_{\rho\omega}(s)$ into two contribution of the direct coupling of $\omega \rightarrow 2\pi$ and the mixing of $\omega \rightarrow \rho \rightarrow 2\pi$. The emergence of $\tilde{\Pi}_{\rho\omega}(s)$ arises from the inclusion of a nonresonant contribution to $\omega \rightarrow 2\pi$. The appearance of the ρ and ω resonance is associated with complex strong phase from relatively broad ρ resonance region. Especially, there is perhaps larger strong phase from double ρ and ω interference. The *CP* violation origins from the weak phase difference and the strong phase difference. Hence, the decay process of $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ is a great candidate for studying the origin of the *CP* violation.

* Email: ganglv66@sina.com

† Email: luye189@163.com

Meanwhile, it is known that the CP violation is extremely tiny from the pure annihilation decay process in experiment. There is relatively large error in dealing with the decay amplitudes from the QCD factorization approach [7]. The perturbative QCD (PQCD) factorization approach [8–11] is based on k_T factorization. The amplitude can be divided into the convolution of the Wilson coefficients, the light cone wave function, and hard kernels by the low energy effective Hamiltonian. The endpoint singularity can be eliminated by introducing the transverse momentum. However, The transverse momentum integration leads to the double logarithm term which is resummed into the Sudakov form factor. The nonperturbative dynamics are included in the meson wave function which can be extracted from experiment. The hard one can be calculated by perturbation theory.

The remainder of this paper is organized as follows. In Sec. II we present the form of the effective Hamiltonian. In Sec. III we give the calculating formalism and calculation details of CP violation from $\rho - \omega$ mixing in the $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ decay. In Sec. IV we show input parameters. We present the numerical results in Sec. V. Summary and discussion are included in Sec. VI. The related function defined in the text are given in the Appendix.

II. THE EFFECTIVE HAMILTONIAN

With the operator product expansion, the effective weak Hamiltonian can be written as [12]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{uq}^* \left[C_1(\mu)Q_1^u(\mu) + C_2(\mu)Q_2^u(\mu) \right] - V_{tb}V_{tq}^* \left[\sum_{i=3}^{10} C_i(\mu)Q_i(\mu) \right] \right\} + \text{H.c.}, \quad (1)$$

where $q = (d, s)$, G_F represents Fermi constant, C_i ($i=1, \dots, 10$) are the Wilson coefficients, $V_{q_1 q_2}$ (q_1 and q_2 represent quarks) is the CKM matrix element, and O_i is the four quark operator. The operators O_i have the following forms:

$$\begin{aligned} O_1^u &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \\ O_2^u &= \bar{d} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma^\mu (1 - \gamma_5) b, \\ O_3 &= \bar{d} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\ O_4 &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \\ O_5 &= \bar{d} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\ O_6 &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_7 &= \frac{3}{2} \bar{d} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\ O_8 &= \frac{3}{2} \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_9 &= \frac{3}{2} \bar{d} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\ O_{10} &= \frac{3}{2} \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \end{aligned} \quad (2)$$

where α and β are color indices, and $q' = u, d, s, c$ or b quarks. In Eq.(2) O_1^u and O_2^u are tree operators, O_3 – O_6 are QCD penguin operators and O_7 – O_{10} are the operators associated with electroweak penguin diagrams. $C_i(m_b)$ can be written [11],

$$\begin{aligned} C_1 &= -0.2703, & C_2 &= 1.1188, \\ C_3 &= 0.0126, & C_4 &= -0.0270, \\ C_5 &= 0.0085, & C_6 &= -0.0326, \\ C_7 &= 0.0011, & C_8 &= 0.0004, \\ C_9 &= -0.0090, & C_{10} &= 0.0022. \end{aligned} \quad (3)$$

So, we can obtain numerical values of a_i . The combinations a_i of Wilson coefficients are defined as usual [9]:

$$\begin{aligned} a_1 &= C_2 + C_1/3, & a_2 &= C_1 + C_2/3, \\ a_3 &= C_3 + C_4/3, & a_4 &= C_4 + C_3/3, \\ a_5 &= C_5 + C_6/3, & a_6 &= C_6 + C_5/3, \\ a_7 &= C_7 + C_8/3, & a_8 &= C_8 + C_7/3, \\ a_9 &= C_9 + C_{10}/3, & a_{10} &= C_{10} + C_9/3. \end{aligned} \quad (4)$$

III. CP VIOLATION IN $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$

A. Formalism

The amplitudes A^σ of the process $\bar{B}_s(p) \rightarrow V_1(p_1, \epsilon_1) + V_2(p_2, \epsilon_2)$ can be written [13]

$$A^\sigma = \epsilon_{1\mu}^*(\sigma)\epsilon_{2\nu}^*(\sigma)(ag^{\mu\nu} + \frac{b}{m_1m_2}p^\mu p^\nu + \frac{ic}{m_1m_2}\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}) \quad (5)$$

where σ is the helicity of the vector meson. $\epsilon_1(p_1)$ and $\epsilon_2(p_2)$ are the polarization vectors (momenta) of V_1 and V_2 , respectively. m_1 and m_2 refer to the masses of the vector mesons V_1 and V_2 . The invariant amplitudes a, b, c are associated with the amplitude A_i (i refer to the three kind of polarizations, longitudinal (L), normal (N) and transverse (T)). Then we have

$$A^\sigma = M_{B_s}^2 A_L + M_{B_s}^2 A_N \epsilon_{1\mu}^*(\sigma = T) \cdot \epsilon_{2\mu}^*(\sigma = T) + iA_T \epsilon^{\alpha\beta\gamma\rho} \epsilon_{1\alpha}^*(\sigma) \epsilon_{2\beta}^*(\sigma) p_{1\gamma} p_{2\rho} \quad (6)$$

The longitudinal H_0 , transverse H_\pm of helicity amplitudes can be expressed $H_0 = M_{B_s}^2 A_L$, $H_\pm = M_{B_s}^2 A_N \mp m_1 m_2 \sqrt{r^2 - 1} A_T$. The decay width is written

$$\Gamma = \frac{P_c}{8\pi M_{B_s}^2} A^{(\sigma)+} A^{(\sigma)} = \frac{P_c}{8\pi M_{B_s}^2} |H_0|^2 + |H_+|^2 + |H_-|^2. \quad (7)$$

The interaction of the photon and the hadronic matter can be described by the vector meson dominance model

(VMD) [14]. The photon can couple to the hadronic field through a ρ meson. The mixing matrix element $\tilde{\Pi}_{\rho\omega}(s)$ is extracted from the data of the cross section for $e^+e^- \rightarrow \pi^+\pi^-$ [15, 16]. The nonresonant contribution of $\omega \rightarrow \pi^+\pi^-$ has been effectively absorbed into $\tilde{\Pi}_{\rho\omega}$ which leads to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$ [17]. We can make the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$. However, one can neglect the s dependence of $\tilde{\Pi}_{\rho\omega}$ in practice. The $\rho - \omega$ mixing parameters were determined in the fit of Gardner and O'Connell [18]:

$$\begin{aligned}\Re\tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -3500 \pm 300\text{MeV}^2, \\ \Im\tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -300 \pm 300\text{MeV}^2, \\ \tilde{\Pi}'_{\rho\omega}(m_\omega^2) &= 0.03 \pm 0.04.\end{aligned}\tag{8}$$

The formalism of the CP violation is presented for the \bar{B}_s^0 meson decay process in the following. The amplitude A (\bar{A}) for the decay process $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ ($B_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$) can be written as:

$$A = \langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}_s^0 \rangle + \langle \pi^+\pi^-\pi^+\pi^- | H^P | \bar{B}_s^0 \rangle,\tag{9}$$

$$\bar{A} = \langle \pi^+\pi^-\pi^+\pi^- | H^T | B_s^0 \rangle + \langle \pi^+\pi^-\pi^+\pi^- | H^P | B_s^0 \rangle,\tag{10}$$

where H^T and H^P refer to the tree and penguin operators in the Hamiltonian, respectively. We define the relative magnitudes and phases between the tree and penguin operator contributions as follows:

$$A = \langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}_s^0 \rangle [1 + re^{i(\delta+\phi)}],\tag{11}$$

$$\bar{A} = \langle \pi^+\pi^-\pi^+\pi^- | H^T | B_s^0 \rangle [1 + re^{i(\delta-\phi)}],\tag{12}$$

where δ and ϕ are strong and weak phases, respectively. The weak phase difference ϕ can be expressed as a combination of the CKM matrix elements: $\phi = \arg[(V_{tb}V_{ts}^*)/(V_{ub}V_{us}^*)]$. The parameter r is the absolute value of the ratio of tree and penguin amplitudes:

$$r \equiv \left| \frac{\langle \pi^+\pi^-\pi^+\pi^- | H^P | \bar{B}_s^0 \rangle}{\langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}_s^0 \rangle} \right|.\tag{13}$$

The parameter of CP violating asymmetry, A_{CP} , can be written as

$$A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2(T_0^2 r_0 \sin \delta_0 + T_+^2 r_+ \sin \delta_+ + T_-^2 r_- \sin \delta_-) \sin \phi}{\sum_{i=0+-} T_i^2 (1 + r_i^2 + 2r_i \cos \delta_i \cos \phi)},\tag{14}$$

where

$$|A|^2 = \sum_{\sigma} A^{(\sigma)+} A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2\tag{15}$$

and T_i ($i = 0, +, -$) represent the tree-level helicity amplitudes. We can see explicitly from Eq. (14) that both weak

and strong phase differences are responsible for CP violation. $\rho - \omega$ mixing introduces the strong phase difference and well known in the three body decay processes of the bottom hadron [19–25]. Due to $\rho - \omega$ interference from the u and d quark mixing, we can write the following formalism in an approximate from the first order of isospin violation:

$$\langle \pi^+ \pi^- \pi^+ \pi^- | H^T | \bar{B}_s^0 \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega} \tilde{\Pi}_{\rho\omega} t_{\rho\omega} + \frac{g_\rho^2}{s_\rho^2} t_{\rho\rho}, \quad (16)$$

$$\langle \pi^+ \pi^- \pi^+ \pi^- | H^P | \bar{B}_s^0 \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega} \tilde{\Pi}_{\rho\omega} p_{\rho\omega} + \frac{g_\rho^2}{s_\rho^2} p_{\rho\rho}, \quad (17)$$

where $t_{\rho\rho}(p_{\rho\rho})$ and $t_{\rho\omega}(p_{\rho\omega})$ are the tree (penguin) amplitudes for $\bar{B}_s \rightarrow \rho^0 \rho^0$ and $\bar{B}_s \rightarrow \rho^0 \omega$, respectively, g_ρ is the coupling for $\rho^0 \rightarrow \pi^+ \pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude which also effectively includes the direct coupling $\omega \rightarrow \pi^+ \pi^-$. s_V , m_V and Γ_V ($V = \rho$ or ω) is the inverse propagator, mass and decay rate of the vector meson V , respectively.

$$s_V = s - m_V^2 + im_V \Gamma_V, \quad (18)$$

with \sqrt{s} being the invariant masses of the $\pi^+ \pi^-$ pairs. There are double $\rho - \omega$ interference in the decay process of $\bar{B}_s^0 \rightarrow \rho^0(\omega) \rho^0(\omega) \rightarrow \pi^+ \pi^- \pi^+ \pi^-$. Hence, a factor of 2 appears in Eqs. (16), (17) compared with the case of single $\rho - \omega$ interference [19–27]. From Eqs. (9)(11)(16)(17) one has

$$r e^{i\delta} e^{i\phi} = \frac{2\tilde{\Pi}_{\rho\omega} p_{\rho\omega} + s_\omega p_{\rho\rho}}{2\tilde{\Pi}_{\rho\omega} t_{\rho\omega} + s_\omega t_{\rho\rho}}, \quad (19)$$

Defining

$$\frac{p_{\rho\omega}}{t_{\rho\rho}} \equiv r' e^{i(\delta_q + \phi)}, \quad \frac{t_{\rho\omega}}{t_{\rho\rho}} \equiv \alpha e^{i\delta_\alpha}, \quad \frac{p_{\rho\rho}}{p_{\rho\omega}} \equiv \beta e^{i\delta_\beta}, \quad (20)$$

where δ_α , δ_β and δ_q are strong phases, one finds the following expression from Eqs. (19)(20):

$$r e^{i\delta} = r' e^{i\delta_q} \frac{2\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{2\tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha} + s_\omega}. \quad (21)$$

In order to obtain the CP violating asymmetry in Eq. (14), $\sin\phi$ and $\cos\phi$ are needed, where ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [28], one has

$$\begin{aligned} \sin\phi &= -\frac{\eta}{\sqrt{\rho^2 + \eta^2}}, \\ \cos\phi &= -\frac{\rho}{\sqrt{\rho^2 + \eta^2}}. \end{aligned} \quad (22)$$

B. Calculation details

We can decompose the decay amplitude for the decay process $\bar{B}_s^0 \rightarrow \rho^0(\omega) \rho^0(\omega)$ in terms of tree-level and penguin-level contributions depending on the CKM matrix elements of $V_{ub}V_{us}^*$ and $V_{tb}V_{ts}^*$. Due to the equations (14)(19)(20),

we calculate the amplitudes $t_{\rho\rho}$, $t_{\rho\omega}$, $p_{\rho\rho}$ and $p_{\rho\omega}$ in perturbative QCD approach. The F and M function associated with the decay amplitudes can be found in the appendix from the perturbative QCD approach.

There are four types of Feynman diagrams contributing to $\bar{B}_s \rightarrow M_2 M_3 (M_2, M_3 = \rho \text{ or } \omega)$ annihilation decay mode at leading order. The pure annihilation type process can be classified into factorizable diagrams and non-factorizable diagrams [29, 30]. Through calculating these diagrams, we can get the amplitudes $A^{(i)}$, where $i = L, N, T$ standing for the longitudinal and two transverse polarizations. Because these diagrams are the same as those of $B \rightarrow K^* \phi$ and $B \rightarrow K^* \rho$ decays [29, 30], the formulas of $\bar{B}_s \rightarrow \rho\rho$ or $\bar{B}_s \rightarrow \rho\omega$ are similar to those of $B \rightarrow K^* \phi$ and $B \rightarrow K^* \rho$. We just need to replace some corresponding wave functions, Wilson coefficients and corresponding parameters.

With the Hamiltonian (1), depending on CKM matrix elements of $V_{ub}V_{us}^*$ and $V_{tb}V_{ts}^*$, the decay amplitudes $A^{(i)} (i = L, N, T)$ for $\bar{B}_s^0 \rightarrow \rho^0 \rho^0$ in PQCD can be written as

$$\sqrt{2}A^{(i)}(\bar{B}_s^0 \rightarrow \rho^0 \rho^0) = V_{ub}V_{us}^* t_{\rho\rho}^i - V_{tb}V_{ts}^* p_{\rho\rho}^i, \quad (23)$$

The tree level amplitude $t_{\rho\rho}$ can be written as

$$t_{\rho\rho}^i = \frac{G_F}{\sqrt{2}} \left\{ f_{B_s} F_{ann}^{LL,i} [a_2] + M_{ann}^{LL,i} [C_2] \right\}, \quad (24)$$

where f_{B_s} refers to the decay constant of \bar{B}_s meson.

The penguin level amplitude are expressed in the following

$$\begin{aligned} p_{\rho\rho}^i = \frac{G_F}{\sqrt{2}} \left\{ f_{B_s} F_{ann}^{LL,i} \left[2a_3 + \frac{1}{2}a_9 \right] + f_{B_s} F_{ann}^{LR,i} \left[2a_5 + \frac{1}{2}a_7 \right] \right. \\ \left. + M_{ann}^{LL,i} \left[2C_4 + \frac{1}{2}C_{10} \right] + M_{ann}^{SP,i} \left[2C_6 + \frac{1}{2}C_8 \right] \right\}. \end{aligned} \quad (25)$$

The decay amplitude for $\bar{B}_s^0 \rightarrow \rho^0 \omega$ can be written as

$$2A^{(i)}(\bar{B}_s^0 \rightarrow \rho^0 \omega) = V_{ub}V_{us}^* t_{\rho\omega}^i - V_{tb}V_{ts}^* p_{\rho\omega}^i. \quad (26)$$

We can give the tree level the contribution in the following

$$t_{\rho\omega}^i = \frac{G_F}{\sqrt{2}} \left\{ f_{B_s} F_{ann}^{LL,i} [a_2] + M_{ann}^{LL,i} [C_2] \right\}, \quad (27)$$

and the penguin level contribution are given as following

$$\begin{aligned} p_{\rho\omega}^i = \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left\{ f_{B_s} F_{ann}^{LL,i} \left[\frac{3}{2}a_9 \right] + f_{B_s} F_{ann}^{LR,i} \left[\frac{3}{2}a_7 \right] \right. \\ \left. + M_{ann}^{LL,i} \left[\frac{3}{2}C_{10} \right] + M_{ann}^{SP,i} \left[\frac{3}{2}C_8 \right] \right\} + [\rho^0 \leftrightarrow \omega]. \end{aligned} \quad (28)$$

Based on the definition of (20), we can get

$$\alpha e^{i\delta_\alpha} = \frac{t_{\rho\omega}}{t_{\rho\rho}}, \quad (29)$$

$$\beta e^{i\delta_\beta} = \frac{p_{\rho\rho}}{p_{\rho\omega}}, \quad (30)$$

$$r' e^{i\delta_q} = \frac{p_{\rho\omega}}{t_{\rho\rho}} \times \left| \frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*} \right|, \quad (31)$$

where

$$\left| \frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*} \right| = \frac{\sqrt{\rho^2 + \eta^2}}{\lambda^2(\rho^2 + \eta^2)}. \quad (32)$$

IV. INPUT PARAMETERS

The CKM matrix, which elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters A , ρ , λ and η [28]:

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (33)$$

where $\mathcal{O}(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [31]:

$$\begin{aligned} \lambda &= 0.22537 \pm 0.00061, \quad A = 0.814_{-0.024}^{+0.023}, \\ \bar{\rho} &= 0.117 \pm 0.21, \quad \bar{\eta} = 0.353 \pm 0.013. \end{aligned} \quad (34)$$

where

$$\bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \quad (35)$$

From Eqs. (34) (35) we have

$$0.121 < \rho < 0.158, \quad 0.336 < \eta < 0.363. \quad (36)$$

The other parameters and the corresponding references are listed in Table.1.

V. THE NUMERICAL RESULTS OF CP VIOLATION IN $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$

In the numerical results, we find that the CP violation can be enhanced via double $\rho - \omega$ mixing for the pure annihilation type decay channel $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance within perturbative QCD scheme. The CP violation depends on the weak phase difference from

TABLE I: Input parameters used in this paper.

Parameters	Input data	References
Fermi constant (in GeV^{-2})	$G_F = 1.16638 \times 10^{-5}$	[32]
Masses and decay widths (in GeV)	$m_{B_s^0} = 5.36677$, $\tau_{B_s^0} = 1.512 \times 10^{-12} s$ $m_{\rho^0(770)} = 0.77526$, $\Gamma_{\rho^0(770)} = 0.1491$, $m_{\omega(782)} = 0.78265$, $\Gamma_{\omega(782)} = 8.49 \times 10^{-3}$, $m_\pi = 0.13957$, $m_W = 80.385$, $m_u = 0.0023$, $m_d = 0.0048$, $m_s = 0.095$, $m_c = 1.275$, $m_t = 173.21$, $m_b = 4.18$,	[32]
Decay constants (in MeV)	$f_\rho = 209 \pm 2$, $f_\rho^T = 165 \pm 9$, $f_\omega = 195.1 \pm 3$, $f_\omega^T = 145 \pm 10$,	[32, 33]

CKM matrix elements and the strong phase difference which is difficult to control. The CKM matrix elements, which relate to ρ , η , λ and A , are given in Eq.(34). The uncertainties due to the CKM matrix elements come from ρ , η , λ and A . In our numerical calculations, we let ρ , η , λ and A vary among the limiting values. The numerical results are shown from Fig. 1 to Fig. 3 with the different parameter values of CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$, respectively. We find the results are not sensitive to the values of ρ , η , λ and A . In Fig. 1, we give the plot of CP violating asymmetry as a function of \sqrt{s} . From the Fig. 1, one can see the CP violation parameter is dependent on \sqrt{s} and changes rapidly due to $\rho - \omega$ mixing when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance. From the numerical results, it is found that the maximum CP violating parameter reaches 28.64% in the case of $(\rho_{mini}, \eta_{mini})$.

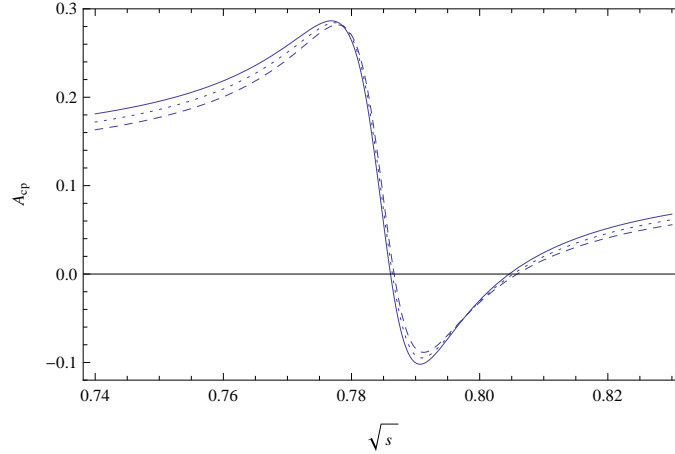


FIG. 1: The CP violating asymmetry, A_{cp} , as a function of \sqrt{s} for different CKM matrix elements. The dash line, dot line and solid line corresponds to the maximum, middle, and minimum CKM matrix element for the decay channel of $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$, respectively.

From Eq.(14), one can see that the CP violating parameter depend on both $\sin\delta$ and r . The plots of $\sin\delta$ and r as a function of \sqrt{s} are shown in Fig. 2, and Fig. 3, respectively. It can be seen that $\sin\delta_0$ ($\sin\delta_-$ and $\sin\delta_+$) vary sharply at the range of the resonance in Fig. 2. One can see that r change largely in the vicinity of the ω resonance.

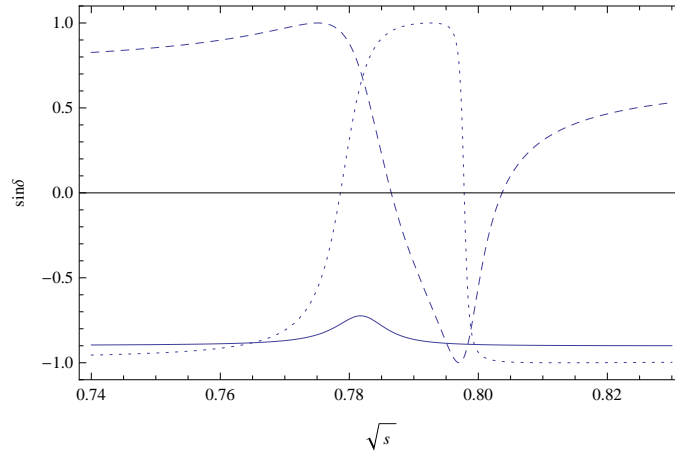


FIG. 2: $\sin\delta$ as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. The dash line, dot line and solid line corresponds to $\sin\delta_0$, $\sin\delta_+$ and $\sin\delta_-$, respectively.

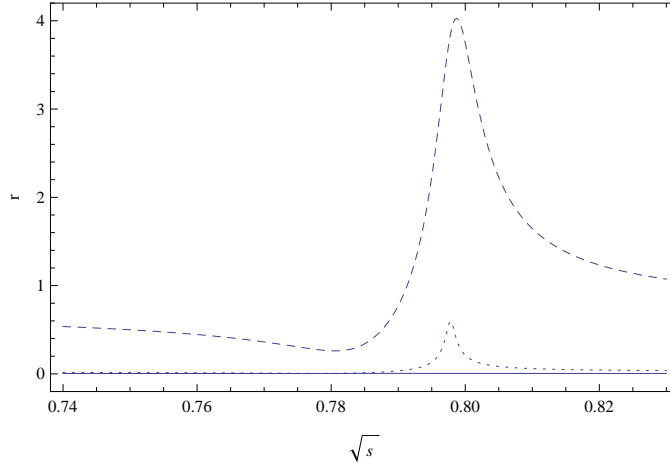


FIG. 3: Plot of r as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. The dash line, dot line and solid line corresponds to r_0 , r_+ and r_- , respectively.

VI. SUMMARY AND CONCLUSION

In this paper, we study the CP violation for the pure annihilation type decay process of $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ in perturbative QCD. It has been found that the CP violation can be enhanced greatly at the area of $\rho - \omega$ resonance. The maximum CP violation value can reach 28.64% due to double ρ and ω resonance.

The theoretical errors are large which follows to the uncertainties of results. Generally, power corrections beyond the heavy quark limit give the major theoretical uncertainties. This implies the necessity of introducing $1/m_b$ power corrections. Unfortunately, there are many possible $1/m_b$ power suppressed effects and they are generally nonperturbative in nature and hence not calculable by the perturbative method. There are more uncertainties in this scheme. The first error refers to the variation of the CKM parameters, which are given in Eq.(34). The second error comes from the hadronic parameters: the shape parameters, form factors, decay constants, and the wave function of the B_s meson. The third error corresponds to the choice of the hard scales, which vary from 0.75t to 1.25t, which character-

izing the size of next-to-leading order QCD contributions. Therefore, the results for CP violating asymmetries of the decay process $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ is given as following:

$$A_{CP}(\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-) = 28.43_{-0.25-0.16-3.98}^{+0.21+0.25+5.62}\%, \quad (37)$$

where the first uncertainty is corresponding to the CKM parameters, the second comes from the hadronic parameters, and the third is associated with the hard scales. The LHC experiment may detect the large CP violation for the decay process $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ in the region of the ω resonance.

VII. APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

In this appendix we present explicit expressions of the factorizable and non-factorizable amplitudes with Perturbative QCD in Eq.(23) and Eq.(26) [10, 11, 34, 35]. The factorizable amplitudes $F_{ann}^{LL,i}(a_i)$, and $F_{ann}^{SP,i}(a_i)$ (i=L,N,T) are written as

$$f_{B_s} F_{ann}^{LL,N}(a_i) = f_{B_s} F_{ann}^{LR,N}(a_i) \quad (38)$$

$$\begin{aligned} f_{B_s} F_{ann}^{LL,N}(a_i) = & -8\pi C_F M_{B_s}^4 f_{B_s} r_2 r_3 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ E_a(t_c) a_i(t_c) h_a(x_2, 1-x_3, b_2, b_3) \right. \\ & [(2-x_3)(\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) + x_3(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))] \\ & - h_a(1-x_3, x_2, b_3, b_2)[(1+x_2)(\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) \\ & \left. - (1-x_2)(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))] E_a(t'_c) a_i(t'_c) \right\}. \end{aligned} \quad (39)$$

$$f_{B_s} F_{ann}^{LL,T}(a_i) = -f_{B_s} F_{ann}^{LR,T}(a_i) \quad (40)$$

$$\begin{aligned} f_{B_s} F_{ann}^{LL,T}(a_i) = & -16\pi C_F M_{B_s}^4 f_{B_s} r_2 r_3 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ [x_3(\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) \right. \\ & + (2-x_3)(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))] E_a(t_c) a_i(t_c) h_a(x_2, 1-x_3, b_2, b_3) \\ & + h_a(1-x_3, x_2, b_3, b_2)[(1-x_2)(\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) \\ & \left. - (1+x_2)(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))] E_a(t'_c) a_i(t'_c) \right\}. \end{aligned} \quad (41)$$

$$\begin{aligned}
f_{B_s} F_{ann}^{LL,L}(a_i) = & 8\pi C_F M_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ a_i(t_c) E_a(t_c) \right. \\
& \times \left[(x_3 - 1) \phi_2(x_2) \phi_3(x_3) - 4r_2 r_3 \phi_2^s(x_2) \phi_3^s(x_3) \right. \\
& + 2r_2 r_3 x_3 \phi_2^s(x_2) (\phi_3^s(x_3) - \phi_3^t(x_3)) \left. \right] h_a(x_2, 1 - x_3, b_2, b_3) \\
& + \left[x_2 \phi_2(x_2) \phi_3(x_3) + 2r_2 r_3 (\phi_2^s(x_2) - \phi_2^t(x_2)) \phi_3^s(x_3) \right. \\
& + 2r_2 r_3 x_2 (\phi_2^s(x_2) + \phi_2^t(x_2)) \phi_3^s(x_3) \left. \right] a_i(t'_c) E_a(t'_c) h_a(1 - x_3, x_2, b_3, b_2) \left. \right\}. \tag{42}
\end{aligned}$$

$$F_{ann}^{LR,L}(a_i) = F_{ann}^{LL,L}(a_i), \tag{43}$$

with the color factor $C_F = 3/4$, f_{B_s} refer to the decay constant of \bar{B}_s meson and a_i represents the corresponding Wilson coefficients for annihilation decay channels. In the above functions, $r_2(r_3) = m_V/m_{B_s}$ and $\phi_2(\phi_3) = \phi_V$ ($V = \rho$ or ω), where m_V is the chiral scale parameter.

The non-factorizable amplitudes $M_{ann}^{LL,i}(a_i)$, and $M_{ann}^{SP,i}(a_i)$ (i=L,N,T) are written as

$$M_{ann}^{LL,N}(a_i) = M_{ann}^{SP,N}(a_i) \tag{44}$$

$$\begin{aligned}
M_{ann}^{LL,N}(a_i) = & -64\pi C_F M_{B_s}^4 r_2 r_3 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_2 b_2 db_2 \phi_{B_s}(x_1, b_1) [\phi_2^v(x_2) \phi_3^v(x_3) \\
& + \phi_2^a(x_2) \phi_3^a(x_3)] E'_a(t_d) a_i(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2), \tag{45}
\end{aligned}$$

$$M_{ann}^{LL,T}(a_i) = -M_{ann}^{SP,T}(a_i) \tag{46}$$

$$\begin{aligned}
M_{ann}^{LL,T}(a_i) = & -128\pi C_F M_{B_s}^4 r_2 r_3 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_2 b_2 db_2 \phi_{B_s}(x_1, b_1) [\phi_2^v(x_2) \phi_3^a(x_3) \\
& + \phi_2^a(x_2) \phi_3^v(x_3)] E'_a(t_d) a_i(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2), \tag{47}
\end{aligned}$$

$$\begin{aligned}
M_{ann}^{LL,L}(a_i) = & 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\
& \times \left\{ h_{na}(x_1, x_2, x_3, b_1, b_2) \left[-x_2 \phi_2(x_2) \phi_3(x_3) - 4r_2 r_3 \phi_2^s(x_2) \phi_3^s(x_3) \right. \right. \\
& + r_2 r_3 (1-x_2) (\phi_2^s(x_2) + \phi_2^t(x_2)) (\phi_3^s(x_3) - \phi_3^t(x_3)) \\
& + r_2 r_3 x_3 (\phi_2^s(x_2) - \phi_2^t(x_2)) (\phi_3^s(x_3) + \phi_3^t(x_3)) \left. \right] a_i(t_d) E'_a(t_d) \\
& + h'_{na}(x_1, x_2, x_3, b_1, b_2) \left[(1-x_3) \phi_2(x_2) \phi_3(x_3) \right. \\
& + (1-x_3) r_2 r_3 (\phi_2^s(x_2) + \phi_2^t(x_2)) (\phi_3^s(x_3) - \phi_3^t(x_3)) \\
& + x_2 r_2 r_3 (\phi_2^s(x_2) - \phi_2^t(x_2)) (\phi_3^s(x_3) + \phi_3^t(x_3)) \left. \right] a_i(t'_d) E'_a(t'_d) \left. \right\}, \tag{48}
\end{aligned}$$

$$\begin{aligned}
M_{ann}^{SP,L}(a_i) = & 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\
& \times \left\{ a_i(t_d) E'_a(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2) \left[(x_3-1) \phi_2(x_2) \phi_3(x_3) \right. \right. \\
& - 4r_2 r_3 \phi_2^s(x_2) \phi_3^s(x_3) + r_2 r_3 x_3 (\phi_2^s(x_2) + \phi_2^t(x_2)) (\phi_3^s(x_3) - \phi_3^t(x_3)) \\
& + r_2 r_3 (1-x_2) (\phi_2^s(x_2) - \phi_2^t(x_2)) (\phi_3^s(x_3) + \phi_3^t(x_3)) \left. \right] \\
& + a_i(t'_d) E'_a(t'_d) h'_{na}(x_1, x_2, x_3, b_1, b_2) \left[x_2 \phi_2(x_2) \phi_3(x_3) \right. \\
& + x_2 r_2 r_3 (\phi_2^s(x_2) + \phi_2^t(x_2)) (\phi_3^s(x_3) - \phi_3^t(x_3)) \\
& + r_2 r_3 (1-x_3) (\phi_2^s(x_2) - \phi_2^t(x_2)) (\phi_3^s(x_3) + \phi_3^t(x_3)) \left. \right] \left. \right\}. \tag{49}
\end{aligned}$$

The hard scale t are chosen as the maximum of the virtuality of the internal momentum transition in the hard amplitudes, including $1/b_i$:

$$t_a = \max\{\sqrt{x_3} M_{B_s}, 1/b_1, 1/b_3\}, \tag{50}$$

$$t'_a = \max\{\sqrt{x_1} M_{B_s}, 1/b_1, 1/b_3\}, \tag{51}$$

$$t_b = \max\{\sqrt{x_1 x_3} M_{B_s}, \sqrt{|1-x_1-x_2| x_3} M_{B_s}, 1/b_1, 1/b_2\}, \tag{52}$$

$$t'_b = \max\{\sqrt{x_1 x_3} M_{B_s}, \sqrt{|x_1-x_2| x_3} M_{B_s}, 1/b_1, 1/b_2\}, \tag{53}$$

$$t_c = \max\{\sqrt{1-x_3} M_{B_s}, 1/b_2, 1/b_3\}, \tag{54}$$

$$t'_c = \max\{\sqrt{x_2} M_{B_s}, 1/b_2, 1/b_3\}, \tag{55}$$

$$t_d = \max\{\sqrt{x_2(1-x_3)} M_{B_s}, \sqrt{1-(1-x_1-x_2)x_3} M_{B_s}, 1/b_1, 1/b_2\}, \tag{56}$$

$$t'_d = \max\{\sqrt{x_2(1-x_3)} M_{B_s}, \sqrt{|x_1-x_2|(1-x_3)} M_{B_s}, 1/b_1, 1/b_2\}. \tag{57}$$

The hard functions h are written as [36]

$$h_e(x_1, x_3, b_1, b_3) = [\theta(b_1 - b_3)I_0(\sqrt{x_3}M_{B_s}b_3)K_0(\sqrt{x_3}M_{B_s}b_1) + \theta(b_3 - b_1)I_0(\sqrt{x_3}M_{B_s}b_1)K_0(\sqrt{x_3}M_{B_s}b_3)] K_0(\sqrt{x_1x_3}M_{B_s}b_1)S_t(x_3), \quad (58)$$

$$h_n(x_1, x_2, x_3, b_1, b_2) = [\theta(b_2 - b_1)K_0(\sqrt{x_1x_3}M_{B_s}b_2)I_0(\sqrt{x_1x_3}M_{B_s}b_1) + \theta(b_1 - b_2)K_0(\sqrt{x_1x_3}M_{B_s}b_1)I_0(\sqrt{x_1x_3}M_{B_s}b_2)] \times \begin{cases} \frac{i\pi}{2}H_0^{(1)}(\sqrt{(x_2 - x_1)x_3}M_{B_s}b_2), & x_1 - x_2 < 0 \\ K_0(\sqrt{(x_1 - x_2)x_3}M_{B_s}b_2), & x_1 - x_2 > 0 \end{cases}, \quad (59)$$

$$h_a(x_2, x_3, b_2, b_3) = (\frac{i\pi}{2})^2 S_t(x_3) [\theta(b_2 - b_3)H_0^{(1)}(\sqrt{x_3}M_{B_s}b_2)J_0(\sqrt{x_3}M_{B_s}b_3) + \theta(b_3 - b_2)H_0^{(1)}(\sqrt{x_3}M_{B_s}b_3)J_0(\sqrt{x_3}M_{B_s}b_2)] H_0^{(1)}(\sqrt{x_2x_3}M_{B_s}b_2), \quad (60)$$

$$h_{na}(x_1, x_2, x_3, b_1, b_2) = \frac{i\pi}{2} [\theta(b_1 - b_2)H_0^{(1)}(\sqrt{x_2(1 - x_3)}M_{B_s}b_1)J_0(\sqrt{x_2(1 - x_3)}M_{B_s}b_2) + \theta(b_2 - b_1)H_0^{(1)}(\sqrt{x_2(1 - x_3)}M_{B_s}b_2)J_0(\sqrt{x_2(1 - x_3)}M_{B_s}b_1)] \times K_0(\sqrt{1 - (1 - x_1 - x_2)x_3}M_{B_s}b_1), \quad (61)$$

$$h'_{na}(x_1, x_2, x_3, b_1, b_2) = \frac{i\pi}{2} [\theta(b_1 - b_2)H_0^{(1)}(\sqrt{x_2(1 - x_3)}M_{B_s}b_1)J_0(\sqrt{x_2(1 - x_3)}M_{B_s}b_2) + \theta(b_2 - b_1)H_0^{(1)}(\sqrt{x_2(1 - x_3)}M_{B_s}b_2)J_0(\sqrt{x_2(1 - x_3)}M_{B_s}b_1)] \times \begin{cases} \frac{i\pi}{2}H_0^{(1)}(\sqrt{(x_2 - x_1)(1 - x_3)}M_{B_s}b_1), & x_1 - x_2 < 0 \\ K_0(\sqrt{(x_1 - x_2)(1 - x_3)}M_{B_s}b_1), & x_1 - x_2 > 0 \end{cases}, \quad (62)$$

where J_0 and Y_0 are the Bessel function with $H_0^{(1)}(z) = J_0(z) + iY_0(z)$.

The threshold re-sums factor S_t follows the parameterized [37]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)}[x(1 - x)]^c, \quad (63)$$

where the parameter $c = 0.4$. In the nonfactorizable contributions, $S_t(x)$ gives a very small numerical effect to the amplitude [38]. Therefore, we drop $S_t(x)$ in h_n and h_{na} .

The evolution factors $E_e^{(t)}$ and $E_a^{(t)}$ entering in the expressions for the matrix elements are given by

$$E_e(t) = \alpha_s(t) \exp[-S_B(t) - S_3(t)], \quad E'_e(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_1=b_3}, \quad (64)$$

$$E_a(t) = \alpha_s(t) \exp[-S_2(t) - S_3(t)], \quad E'_a(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_2=b_3}, \quad (65)$$

in which the Sudakov exponents are defined as

$$S_B(t) = s \left(x_1 \frac{M_{B_s}}{\sqrt{2}}, b_1 \right) + \frac{5}{3} \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \quad (66)$$

$$S_2(t) = s \left(x_2 \frac{M_{B_s}}{\sqrt{2}}, b_2 \right) + s \left((1 - x_2) \frac{M_{B_s}}{\sqrt{2}}, b_2 \right) + 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \quad (67)$$

where $\gamma_q = -\alpha_s/\pi$ is the anomalous dimension of the quark. The explicit form for the function $s(Q, b)$ is:

$$s(Q, b) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1} (\hat{q} - \hat{b}) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{\hat{b}} - 1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right] \ln\left(\frac{\hat{q}}{\hat{b}}\right) \\ + \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q} \left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3} [\ln^2(2\hat{q}) - \ln^2(2\hat{b})], \quad (68)$$

where the variables are defined by

$$\hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \quad \hat{b} \equiv \ln[1/(b\Lambda)], \quad (69)$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1 = \frac{33-2n_f}{12}, \quad \beta_2 = \frac{153-19n_f}{24}, \\ A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{1}{2}e^{\gamma_E}\right), \quad (70)$$

with n_f is the number of the quark flavors and γ_E is the Euler constant. We will use the one-loop expression of the running coupling constant.

In this study, we use the model function

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp\left[-\frac{M_{B_s}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \quad (71)$$

where the share parameter $\omega_b = 0.5 \pm 0.05$ GeV, and the normalization constant $N_{B_s} = 63.5688$ GeV is related to the B_s decay constant $f_{B_s} = 0.23 \pm 0.03$ GeV.

For ρ and ω vector meson, we use $\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ and $\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. The distribution amplitudes of vector meson($v=\rho$ or ω), ϕ_ρ , ϕ_ω , ϕ_V^t , ϕ_V^s , ϕ_V^v , and ϕ_V^a , are calculated using light-cone QCD sum rule [39]:

$$\phi_\rho(x) = \frac{3f_\rho}{\sqrt{6}} x(1-x) \left[1 + 0.15C_2^{3/2}(t)\right], \quad (72)$$

$$\phi_\omega(x) = \frac{3f_\omega}{\sqrt{6}} x(1-x) \left[1 + 0.15C_2^{3/2}(t)\right], \quad (73)$$

$$\phi_V^t(x) = \frac{3f_V^T}{2\sqrt{6}} t^2, \quad (74)$$

$$\phi_V^s(x) = \frac{3f_V^T}{2\sqrt{6}} (-t), \quad (75)$$

$$\phi_V^v(x) = \frac{3f_V}{8\sqrt{6}} (1+t^2), \quad (76)$$

$$\phi_V^a(x) = \frac{3f_V}{4\sqrt{6}} (-t), \quad (77)$$

where $t = 2x - 1$. Here f_V is the decay constant of the vector meson with longitudinal polarization, whose values are shown in table I.

The Gegenbauer polynomials $C_n^\nu(t)$ read,

$$\begin{aligned} C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), & C_4^{1/2}(t) &= \frac{1}{8}(35t^4 - 30t^2 + 3), \\ C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), & C_4^{3/2}(t) &= \frac{15}{8}(1 - 14t^2 + 21t^4), \\ C_1^{3/2}(t) &= 3t. \end{aligned} \tag{78}$$

Acknowledgments

This work was supported by National Natural Science Foundation of China (Project Numbers 11605041), Plan For Scientific Innovation Talent of Henan University of Technology (Project Number 2012CXRC17), the Key Project (Project Number 14A140001) for Science and Technology of the Education Department Henan Province, the Fundamental Research Funds (Project Number 2014YWQN06) for the Henan Provincial Colleges and Universities, and the Research Foundation of the young core teacher from Henan province.

-
- [1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
 - [2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
 - [3] Jussara M. de Miranda on behalf of the LHCb collaboration, Proceedings of CKM 2012, the 7th International Workshop on the CKM unitarity, University of Cincinnati (USA), 28 September- 2 October 2012. arXiv:1301.0283 [hep-ex].
 - [4] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. **112**, 011801 (2014).
 - [5] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D **90**, 112004 (2014).
 - [6] H. Fritzsch and A.S. Müller, Nucl. Phys. Proc. Suppl. **96**, 273 (2001).
 - [7] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914(1999); Nucl. Phys. B **606**, 245 (2001).
 - [8] A. Ali and C. Greub, Phys. Rev. D **57**, 2996 (1998); G. Kramer, W. F. Palmer and H. Simma, Nucl. Phys. B **428**, 77 (1994); Z. Phys. C **66**, 429 (1995).
 - [9] A. Ali, G. Kramer, and C. -D. Lü, Phys. Rev. D **58**, 094009 (1998); Phys. Rev. D **59**, 014005 (1999); Y. H. Chen, H. Y. Cheng, B. Tseng, and K. C. Yang, Phys. Rev. D **60**, 094014 (1999).
 - [10] Y. Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).
 - [11] C.-D. Lü, K. Ukai and M.-Z. Yang, Phys. Rev. D **63**, 074009(2001).
 - [12] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
 - [13] G. Kramer, W. F. Palmer, Phys. Rev. D **45**, 193 (1992).
 - [14] J.J. Sakurai, *Currents and Mesons*, University of Chicago Press (1969).
 - [15] H.B. O'Connell, B.C. Pearce, A.W. Thomas, and A.G. Williams, Prog. Part. Nucl. Phys. **39**, 201 (1997).
 - [16] H.B. O'Connell, Aust. J. Phys. **50**, 255 (1997).
 - [17] H.B. O'Connell, A.W. Thomas, and A.G. Williams, Nucl. Phys. A **623**, 559 (1997); K. Maltman, H.B. O'Connell, and A.G. Williams, Phys. Lett. B **376**, 19 (1996).
 - [18] S. Gardner and H.B. O'Connell, Phys. Rev. D **57**, 2716 (1998).
 - [19] X.-H. Guo and A.W. Thomas, Phys. Rev. D **58**, 096013 (1998).
 - [20] X.-H. Guo, O. Leitner, and A.W. Thomas, Phys. Rev. D **63**, 056012 (2001).
 - [21] X.-H. Guo and A.W. Thomas, Phys. Rev. D **61**, 116009 (2000).
 - [22] O. Leitner, X.-H. Guo, and A.W. Thomas, Eur. Phys. J. C **31**, 215 (2003).

- [23] X.-H. Guo, Gang Lü and Z.-H. Zhang, Eur. Phys. J. C **58**, 223 (2008).
- [24] Gang Lü, Bao-He Yuan, Ke-Wei Wei, Phys. Rev. D **83**, 014002 (2011).
- [25] Gang Lü, Zhen-Hua Zhang, Xiu-Ying Liu and Li-Ying Zhang, Int. J. Mod. Phys. A **26**, 2899 (2011).
- [26] R. Enomoto and M. Tanabashi, Phys. Lett. B **386**, 413 (1996).
- [27] S. Gardner, H.B. O'Connell, and A.W. Thomas, Phys. Rev. Lett. **80**, 1834 (1998).
- [28] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983); Phys. Rev. Lett. **13**, 562 (1964).
- [29] H.-W. Huang, *et.al*, Phys. Rev. D **73**, 014011 (2006).
- [30] H.-n Li, and S. Mishima, Phys. Rev. D **71**, 054025 (2005).
- [31] G. L, S. T. Li and Y. T. Wang, Phys. Rev. D **94**, 034040 (2016).
- [32] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).
- [33] H. -n. Li and S. Mishima, Phys. Rev. D **74**, 094020 (2006).
- [34] Zhou Rui, Zhi-Tian Zou , and Cai-Dian Lü, Phys. Rev. D **86**, 074019 (2012).
- [35] Xin Liu, Zhen-Jun Xiao, and Cai-Dian Lü, Phys. Rev. D **81**, 014022 (2010).
- [36] H.-n. Li, Phys. Rev. D **66**,094010 (2002).
- [37] T. Kurimoto, H. n. Li and A. I. Sanda, Phys. Rev. D **65**, 014007 (2002).
- [38] H.-n. Li and K. Ukai, Phys. Lett. B **555**, 197 (2003).
- [39] P. Ball, R. Zwicky, Phys. Rev. D **71**, 014029 (2005); P. Ball, V.M. Braun, Nucl. Phys. B **543**, 201 (1999).